

Report Chair of Applied Dynamics April 2011 – December 2012



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1 Preface

This report summarises the activities in research and teaching of the Chair of Applied Dynamics at the University of Erlangen-Nuremberg between April 2011 and December 2012. The Chair of Applied Dynamics (Lehrstuhl für Technische Dynamik LTD) started in April 2011 as a new chair at the University of Erlangen-Nuremberg.

Part of LTD is the Independent Junior Research Group in the DFG Emmy Noether Programme 'Simulation and optimal control of the dynamics of multibody systems in biomechanics and robotics' that has been at the University of Kaiserslautern from May 2009 to March 2011. Research topics are situated in the field of computational mechanics, in particular dynamics and applied mathematics with focus on the simulation of human motion (everyday movements and sports) and robot dynamics as well as the optimization and optimal control of their dynamics.

Sigrid Leyendecker



2 Team

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chair holder Prof. Dr.-Ing. habil. Sigrid Leyendecker

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Gwendolyn B. Johnson, California Institute of Technology Pasadena, California 28 August - 24 September 2011



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3 Research

3.1 Emmy Noether Independent Junior Research Group

The Emmy Noether Programme by the German Research Foundation (DFG) supports young researchers in achieving independence at an early stage of their scientific careers. Between May 2009 and March 2011, the Emmy Noether Independent Junior Research Group 'Simulation and optimal control of the dynamics of multibody systems in biomechanics and robotics' has been affiliated with the University of Kaiserslautern consisting of the head (Sigrid Leyendecker) and two PhD students. The Emmy Noether Programme runs for five years in total and the group has been transferred to the University of Erlangen-Nuremberg in April 2011 being now part of the Chair of Applied Dynamics.

3.2 Bionicum

The Bavarian Environment Agency (LfU) (being the central authority for environmental protection and nature conservation, geology and water resources management) has established the centre for bionics 'bionicum' consisting of a visitor's centre in the Tiergarten of the City of Nuremberg with a permanent exhibition and three research projects with a total financial volume of eight million Euro. One of the projects investigates artificial muscles and the research is done in cooperation at the LTD and the Institute for Factory Automation and Production Systems (FAPS) at the University of Erlangen-Nuremberg.

3.3 Cooperation partners

Besides numerous worldwide cooperations with scientists in academia, the LTD is contact with other institutions and industrial partners. The LTD cooperates with the Fraunhofer Institute for Industrial and Economical Mathematics (ITWM) in Kaiserslautern on common interests like biomechanics and nonlinear rod dynamics for wind turbine rotor blades. Furthermore, student theses are supervised together with the Siemens AG and the German Aerospace Center (DLR) concerning the balancing of large medical devices and the generation of optimal walking trajectories.



3.4 Scientific reports

The following pages present a short overview on the various ongoing research projects pursued at the Chair of Applied Dynamics. These are partly financed by third-party funding (German Research Foundation DFG) and in addition by the core support of the university.

Research topics

Numerical experiments for viscoelastic Cosserat rods with Kelvin-Voigt damping Holger Lang, Sigrid Leyendecker, Joachim Linn

Phase lag analysis of variational integrators using interpolation techniques Odysseas T. Kosmas, Sigrid Leyendecker

Computing time investigations of variational multirate systems Tobias Gail, Sigrid Leyendecker

Optimal control of monopedal jumping Michael W. Koch, Sigrid Leyendecker

Asynchronous variational Lie group integration for geometrically exact beam dynamics François M.A. Demoures, François Gay-Balmaz, Thomas Leitz, Sigrid Leyendecker, Sina Ober-Blöbaum, Tudor S. Ratiu

Muscle paths in biomechanical multibody simulations Ramona Maas, Sigrid Leyendecker

A numerical approach to multiobjective optimal control of multibody dynamics Maik Ringkamp, Sigrid Leyendecker, Sina Ober-Blöbaum

Numerical experiments for viscoelastic Cosserat rods with Kelvin-Voigt damping

Holger Lang, Sigrid Leyendecker, Joachim Linn

Although having been known in rational mechanics for several decades, geometrically exact rod models of Cosserat type [1] still provide an interesting topic of research in computational mechanics [2]. Likewise, Cosserat rods provide a useful aproach to model slender flexible structures in multibody system dynamics simulations [3]. The basic kinematics of a Cosserat rod is depicted in Figure 1.

In realistic applications, simulation models for computing the transient response of structural members to dynamic excitations have to account for dissipative effects. In particular, in the case of geometrically exact rods, any approach to model viscous damping requires the inclusion of a frame-indifferent viscoelastic constitutive model already on the level of the continuum formulation of the structural model, such that large displacements and finite rotations can be handled properly.

In our recent work [4], we introduce viscous material damping in our quaternionic reformulation of Simo's rod model [1]. We formulate a Kelvin-Voigt type constitutive model

$$\mathbf{F} = \hat{\mathbb{C}}_F \cdot (\mathbf{V} - \mathbf{V}_0) + \hat{\mathbb{V}}_F \cdot \partial_t \mathbf{V}, \quad \mathbf{M} = \hat{\mathbb{C}}_M \cdot (\mathbf{U} - \mathbf{U}_0) + \hat{\mathbb{V}}_M \cdot \partial_t \mathbf{U}$$
(1)



Figure 1: Left: Kinematic quantities for the (deformed) current and (undeformed) reference configurations of a Cosserat rod. Right: Snapshots of a purely torsional, slightly damped vibration of a cosserat rod, fully clamped at the lower end.

by adding viscous contributions to the material stress resultants $\mathbf{F}(s,t)$ and stress couples $\mathbf{M}(s,t)$. These are assumed to be proportional to the rates $\partial_t \mathbf{U}$ and $\partial_t \mathbf{V}$ of the material strain measures $\mathbf{U}(s,t)$ and $\mathbf{V}(s,t)$ of the rod. In the material constitutive equations (1), the elastic properties of the rod are determined by the effective stiffness parameters contained in the symmetric 3×3 matrices $\hat{\mathbb{C}}_F$ and $\hat{\mathbb{C}}_M$. For homogeneous isotropic materials, both are diagonal and given by

$$\hat{\mathbb{C}}_F = \operatorname{diag}\left(GA\kappa_1, GA\kappa_2, EA\right) , \quad \hat{\mathbb{C}}_M = \operatorname{diag}\left(EI_1, EI_2, GJ_T\right)$$
(2)

with stiffness parameters in terms of the elastic moduli E and G and geometric parameters of the cross section (area A, geometric area moments I_1 and I_2 , torsional area moment $J_T = (I_1 + I_2)\kappa_3$, including modifications by shear correction factors κ_i , i = 1, 2, 3, accounting for out-of-plane cross section warping).

In our recent contribution [5], we present a derivation of the Kelvin-Voigt model (1) from threedimensional continuum theory. In addition to the effective stiffness parameters (2), we derive explicit formulas for the damping parameters of the model given by the diagonal elements of the effective viscosity matrices

$$\hat{\mathbb{V}}_F = \hat{\mathbb{C}}_F \cdot \operatorname{diag}\left(\tau_S, \tau_S, \tau_E\right) \quad , \quad \hat{\mathbb{V}}_M = \hat{\mathbb{C}}_M \cdot \operatorname{diag}\left(\tau_E, \tau_E, \tau_S\right) \tag{3}$$

in terms of the elastic stiffness parameters of the rod and the retardation time constants $\tau_S = \eta_S/G$ and $\tau_E = \eta_E/E$, including the shear and extensional viscosity η_S and η_E , respectively. These damping parameters model the integrated cross-sectional viscous damping behaviour associated to the basic deformation modes (bending, twisting, transverse shearing and extension) of the rod, in the same way as the well known stiffness parameters given above model the corresponding elastic response.

Although the Kelvin-Voigt model with damping parameters as given by (3) is already in use in practical applications [6], provided that reasonable guesses for the choice of the time constants τ_S and τ_E are available, a systematic investigation of the Kelvin-Voigt model has not yet been done for general types of motion and deformation. In this work, we intend to present the results of such a systematic investigation obtained from a variety of numerical experiments, e.g. torsional vibrations in Figure 1 with numerical results in Figure 2.



Figure 2: Left: Dynamic evolution of selected torsional angles for purely torsional vibrations. Right: Energies for damped torsional vibrations. The discrepancy between the total energy E(t) = T(t) + V(t) and $E(t = 0) + \int_0^t P_v(\tau) d\tau$ with the viscous power P_v estimates the numerical damping of the underlying numerical time integration scheme.

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Phase lag analysis of variational integrators using interpolation techniques

Odysseas T. Kosmas, Sigrid Leyendecker

In the general framework of geometric numerical integration, variational integration are very popular, due to their symplectic and momentum conservation properties and their good energy behavior [1]. Following the steps of the derivation of Euler-Lagrange equations in the continuous Lagrangian dynamics, one can derive the discrete time Euler-Lagrange equations. For this purpose, one considers approximate configurations $q_0 \approx q(0)$ and $q_1 \approx q(h)$ and a time step $h \in \mathbb{R}$, in order to replace the parameters of position q and velocity \dot{q} in the continuous time Lagrangian $L: TQ \to \mathbb{R}$ being defined on the tangent bundle to the configuration manifold Q. In the discrete setting, a discrete Lagrangian $L_d: Q \times Q \to \mathbb{R}$ is defined to approximate the action integral along the curve segment between q_k and q_{k+1} , i.e. $L_d(q_k, q_{k+1}) \approx \int_{t_k}^{t_{k+1}} L(q, \dot{q}) dt$. This leads to an action sum $S_d(\gamma_d) = \sum_{k=1}^{N-1} L_d(q_k, q_{k+1})$ with $\gamma_d = (q_0, \ldots, q_N)$ representing the discrete trajectory. The discrete Hamilton's principle states that a motion γ_d of the discrete mechanical system extremizes the action sum, i.e. $\delta S_d = 0$. By differentiation and rearrangement of the terms and having in mind that both q_0 and q_N are fixed, the discrete Euler-Lagrange equations (DEL) are obtained [1, 4]

$$D_2L_d(q_{k-1}, q_k) + D_1L_d(q_k, q_{k+1}) = 0, \qquad k = 1, ..., N-1$$

where the notation $D_i L_d$ indicates the slot derivative with respect to the *i*-th argument of L_d .

The features of the phase fitting technique may be observed in its application to first order ordinary differential equations, as e.g. to the test problem $\frac{dy(t)}{dt} = i\omega y(t)$, y(0) = 1 which has the exact solution of oscillatory type $y(t) = e^{i\omega t}$. A numerical map $\hat{\Phi}(h)$, when applied to a set of known past values, produces a numerical estimation $\hat{y}(t+h)$. By calculating the ratio of the estimated solution to the exact one $\frac{\hat{y}(t+h)}{y(t+h)} = \alpha(\omega h)e^{-i\ell(\omega h)}$, one obtains the phase lag $\ell(\omega h)$ of the numerical map $\hat{\Phi}(h)$. The general goal of the phase fitting technique is to minimize the phase lag while simultaneously forcing $\alpha(\omega h)$ to be as closely as possible to unity, see [3].

To construct high order methods, we approximate the action integral along the curve segment between q_k and q_{k+1} , using a discrete Lagrangian that depends only on the end points. We can obtain expressions for configurations q^j and velocities \dot{q}^j for $j = 0, ..., S, S \in \mathbb{N}$ at time $t^j \in [t_k, t_{k+1}]$ by expressing $t^j = t_k + C^j h$ for $C^j \in [0, 1]$ such that $C^0 = 0, C^S = 1$ for $h = t_{k+1} - t_k$ using

$$q(t^{j}) = g_{1}(t^{j})q_{k} + g_{2}(t^{j})q_{k+1}, \qquad \dot{q}(t^{j}) = \dot{g}_{1}(t^{j})q_{k} + \dot{g}_{2}(t^{j})q_{k+1}$$
(1)

The functions $g_1(t^j)$ and $g_2(t^j)$ are chosen for the intended type of the interpolation, see Table 1. For continuity, $g_1(t_{k+1}) = g_2(t_k) = 0$ and $g_1(t_k) = g_2(t_{k+1}) = 1$ is required.

interpolation	$g_1(t^j)$	$g_2(t^j)$
linear	$1 - rac{t^j - t_k}{h}$	$\frac{t^j - t_k}{h}$
cubic	$1 - \frac{t^j - t_k}{h} - \frac{1}{6} \left[\left(1 - \frac{t^j - t_k}{h} \right)^3 - \left(1 - \frac{t^j - t_k}{h} \right) \right] h^2 \omega^2$	$\frac{t^{j}-t_{k}}{h} - \frac{1}{6} \left[\left(\frac{t^{j}-t_{k}}{h} \right)^{3} - \frac{t^{j}-t_{k}}{h} \right] h^{2} \omega^{2}$
	$\sin\left(u - \frac{t^j - t_k}{h}u\right)$	$\sin\left(\frac{t^j-t_k}{h}u\right)$
trigonometric	$\frac{1}{\sin u}$	$\frac{1}{\sin u}$

Table 1: Functions $g_1(t^j)$ and $g_2(t^j)$ of (1) using linear interpolation, cubic spline interpolation and interpolation via trigonometric functions.

For any choice of interpolation we can now define the discrete Lagrangian by the weighted sum

$$L_d(q_k, q_{k+1}) = h \sum_{j=0}^{S} w_j L(q(t_k + C^j h), \dot{q}(t_k + C^j h)),$$

where it can be easily proved that for maximal algebraic order $\sum_{j=0}^{S} w_j (C^j)^m = \frac{1}{m+1}$, where $m = 0, 1, \ldots$ must hold, see [2].

It can be shown that for the case of variational integrators using trigonometric functions $g_1(t^j)$ and $g_2(t^j)$, the phase lag is zero for $u = \omega h$. For the numerical solution of orbital problems, the estimation of the parameter ω can be obtained by calculating the angular velocity

$$\omega(t) = \frac{|\dot{q}(t) \times \ddot{q}(t)|}{|\dot{q}(t)|^2} \tag{2}$$

for the generalized configuration q(t) of particles motion.

		phase la	$\log \ell(\omega h)$	
frequency	S	linear	cubic	trigonometric
$\omega = 1$	3	-8.33294e - 12	-2.42861e - 17	0
$\omega = 10$	5	-8.29447e - 07	-2.77840e - 10	0
$\omega = 50$	8	-2.30817e - 03	-1.45884e - 06	0

Table 2: Phase lag for integrators using different interpolation techniques for the numerical simulation of the harmonic oscillator with different frequencies ω using h = 0.01. For the trigonometric case $u = \omega h$ has been used, with ω taken from (2).



Figure 1: Total energy evolution for the two body problem using trigonometric interpolation with S = 5 and h = 0.01 compared with ode45, ode113 and ode23. Left: eccentricity $\epsilon = 0.2$. Right: eccentricity $\epsilon = 0.8$.

We first consider the simple harmonic oscillator, see [5], with frequency ω . In Table 2, the calculated phase lag $\ell(\omega h)$ for integrators using linear, cubic and trigonometric interpolation is shown for different frequencies ω using h = 0.01. Moreover, methods with different numbers of intermediate points Shave been used, i.e. more intermediate points are used for cases of high frequencies in order to keep the resulting $\ell(\omega h)$ rather small when using linear and cubic interpolation.

For the case of the planar two body problem in Fig. 1, see [5], the total energy behavior of the proposed trigonometric method is illustrated by plotting the energy evolution for small and high eccentricities, $\epsilon = 0.2$ and $\epsilon = 0.8$, respectively. The results have been compared to standard ode solvers, i.e. ode45, ode113 and ode23 and one can see that the total energy of the proposed technique is stable for long term integration for any chosen eccentricities.

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Computing time investigations of variational multirate systems

Tobias Gail, Sigrid Leyendecker

We investigate the behavior of the computing time of variational mutlirate integration. For mechanical systems with dynamics on varying time scales, the numerical integration has to comply with contradicting requirements. On the one hand, to guarantee a stable integration of the fast motion, we need tiny step sizes. On the other hand, for the slow motions, a larger time step size is accurate enough. Furthermore, too small time steps increase the computing time unnecessarily, especially for costly function evaluations. For this, multirate systems split the system into subsystems [1] which can be solved with different methods [2]. For the multirate scheme we use two time step sizes in the framework described as variational multirate integration [3]. With this approach, we expect less computing time and demonstrate that this is the case by means of numerical examples. The example systems are the Fermi-Pasta-Ulam Problem (FPU), a triple spherical pendulum and a simple atomistic model, all consisting of multiple masspoints and springs while the latter two contain rigid links described by holonmic constraints.

Let a mechanical system be described by a Lagrangian with a configuration vector $q(t) \in Q \subseteq \mathbb{R}^n$ and a velocity vector $\dot{q} \in TQ \subseteq \mathbb{R}^n$. Also, let the mechanical system be constrained by the m^c dimensional holonomic function of constraints requiring g(q) = 0. Now, let the mechanical system contain fast and slow dynamics. Let this be characterized by the possibility to split the variables into n^s slow and n^f fast variables with $q = (q^s, q^f)$ and $n = n^s + n^f$. Furthermore, we assume that we can split the potential energy into a slow potential V(q) and a fast potential $W(q^f)$. Via Hamilton's principal the constrained multirate Euler-Lagrange equations can be derived.

$$\frac{d}{dt}\frac{\partial T}{\partial \dot{q}^s} - \frac{\partial V}{\partial q^s} - \left(\frac{\partial g}{\partial q^s}\right)^T \cdot \lambda = 0$$

$$\frac{d}{dt}\frac{\partial T}{\partial \dot{q}^f} - \frac{\partial V}{\partial q^f} - \frac{\partial W}{\partial q^f} - \left(\frac{\partial g}{\partial q^f}\right)^T \cdot \lambda = 0$$

$$q(q) = 0$$
(1)

Here T denotes the kinetic energy and λ the Lagrangian multiplier. To approximate the solution, rather than choosing one time grid we choose two time grids. Here the

macro time step is ΔT , the micro time step is Δt and $\Delta T \geq \Delta t$ holds.



Figure 1: Example without constraints: the Fermi-Pasta-Ulam Problem with slow and fast variables.



Figure 2: Macro and micro time grid.

The macro time grid provides the domain for the discrete slow variables $q_d^s = \{q_k^s\}_{k=0}^N$ with $q_k^s \approx q^s(t_k)$, while the micro time grid provides the domain for the discrete fast variables $q_d^f = \{\{q_k^{f,m}\}_{m=0}^p\}_{k=0}^{N-1}$ with $q_k^{f,m} \approx q^f(t_k^m)$ and the discrete Lagrangian multipliers $\lambda_d = \{\{\lambda_k^m\}_{m=0}^p\}_{k=0}^{N-1}$ with $\lambda_k^m \approx \lambda(t_k^m)$. With L_d the discrete Lagrangian and h_d the product of discrete constraints and multipliers, we get the

With L_d the discrete Lagrangian and h_d the product of discrete constraints and multipliers, we get the discrete action S_d approximating the continuous action. Via a discrete form of Hamilton's principal requiring stationarity for the discrete action $\delta S_d = 0$, we derive the discrete constrained multirate Euler-Lagrange equations.

To solve this system, a residual Newton-Raphson method with an analytical Jacobian is used. Therefor, we need to derive the discrete multirate Euler-Lagrange equations with respect to all unknowns. For a constrained system the structure of the $(n^s + pn^f + m^{c^s} + pm^{c^{sf}}) \times (n^s + pn^f + m^{c^s} + pm^{c^{sf}})$ Jacobian block matrix is:

$$\begin{bmatrix} D_{q_{k+1}^s}(D_{q_k^s}(L_d+h_d)) & D_{q_k^{f,1,\dots,p}}(D_{q_k^s}(L_d+h_d)) & D_{\lambda_k}(D_{q_k^s}(h_d)) \\ D_{q_{k+1}^s}(D_{q_k^{f,0,\dots,p-1}}(L_d+h_d)) & D_{q_k^{f,1,\dots,p}}(D_{q_k^{f,0,\dots,p-1}}(L_d+h_d)) & D_{\lambda_k}(D_{q_k^{f,0,\dots,p-1}}(h_d)) \\ D_{q_{k+1}^s}(D_{\lambda_k}(h_d)) & D_{q_k^{f,1,\dots,p}}(D_{\lambda_k}(h_d)) & 0 \end{bmatrix}$$
(2)

with p the number of micro steps per macro step and with m^c the number of constraints split into m^{c^s} constraints depending only on slow variables and $m^{c^{sf}}$ constraints depending on slow and fast variables with $m^c = m^{c^s} + m^{c^{sf}}$.

Quadrature rules are needed to approximate the action and constraints by discrete quantities. We use e.g. the midpoint rule, the trapezoidal rule the affine combination and finite difference. Different quadrature rules can be chosen for the kinetic energy, both potential energies and the constraints, which gives us a wide range of possible combinations. The choice of quadrature rule has an influence on the structure of the Jacobian and leads to "fully implicit", "explicit slow, implicit fast" and "fully explicit" schemes.

Since we want to demonstrate that the computing time is lower for an increasing number of micro steps per macro step, we leave the accuracy constant, i.e. a constant micro time step size $\Delta t = const$ is used. This leads to a larger macro time step size with $\Delta T = p\Delta t$. For different numbers of micro steps, within the range $1 \leq p \leq 1000$, we measure the computing times. The computing times are compared for all quadrature rules for all three examples. Figure 4 shows an exemplary plot for the computing times for $1 \leq p \leq 100$ for one example system, the Fermi-Pasta-Ulam Problem, and the different quadrature rules.





Figure 3: Example subject to constraints: triple spherical pendulum with slow and fast variables



The next step is to explain the behavior of the computing times. Therefor, we first look at the total number of iterations and the average number of iterations until the Newton method has converged. Their influence on the computing times for the different example systems is explained. Here, we see that the choice of the quadrature rule has an influence on the average number of iterations. To explain the behavior of the computing time further, we measure the computing times to evaluate the Jacobian and to solve the linear system in one Newton iteration. By measuring the time to store the computed configuration we see the influence on the post processing.

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Optimal control of monopedal jumping

Michael W. Koch, Sigrid Leyendecker

The optimal control of human locomotion requires simulation techniques which handle the contact's establishing and releasing between the foot and the ground. In this work, we consider a monopedal jumper modelled as a three-dimensional constrained multibody system. The investigated contact formulation covers the theory of perfectly plastic contacts. The optimal control problem is solved by a direct transcription method transforming it into a constrained optimisation problem. The applied mechanical integrator is based on a discrete constrained version of the Lagrange-D'Alembert principle, which yields a symplectic momentum preserving method (see [2] for details). To guarantee the structure preservation and the geometrical correctness, we solve the non-smooth problem including the computation of the contact configuration, time and force, in contrast to relying on a smooth approximation of the contact problem via a penalty potential.



Figure 1: Model of the monopedal jumper.

The characteristics of human jumping are analysed using a simplified model consisting of only three rigid bodies representing the human's torso, thigh and calf. The hip is modelled as a spherical joint and the thigh and calf are chained with a revolute joint, where the unit vector n_1 specifies the axis of rotation. In comparison to the monopedal jumper in [3] being supported by a prismatic joint at the upper body, in the jumper model discussed here, this prismatic joint has been removed. Thus, it can perform more human like motion and more general motion. The constrained system of the jumper is described by the configuration variable $q \in \mathbb{R}^{36}$ and due to the rigid body formulation in use, $m_{int} = 18$ internal constraints are present. The anatomical joint interconnections cause $m_{ext} = 8$ external constraints and therefore the k = 36-dimensional system is restricted by m = 26 holonomic constraints. Corresponding to the k - m = 10 degrees of freedom, the generalised coordinates read $\boldsymbol{u} = \begin{bmatrix} \boldsymbol{u}^1; \boldsymbol{\theta}^1; \boldsymbol{\theta}_S; \boldsymbol{\theta}_R \end{bmatrix}^T \in \mathbb{R}^{10}$ and $\boldsymbol{\tau} = \begin{bmatrix} \boldsymbol{\tau}_S; \boldsymbol{\tau}_R \end{bmatrix} \in \mathbb{R}^4$ represents the actuation in the hip and the knee joint (see Figure1 for details). The contact between the foot and the ground is modelled as a perfectly plastic contact, which means, that during the contact phase the foot is fixed at the ground by a spherical joint. As a result of the contact constraints, the degrees of freedom are reduced to k-m-3=7. The contact forces f_S immobilises the jumper's foot and its function is to prevent the penetration of the ground.

The goal of an optimal control problem is to find the optimal trajectory and the optimal control sequence leading the three-dimensional monopedal jumper from an initial to a final state. After the direct transcription, the discrete objective function is to be minimised subject to the reduced discrete equations of motion of the symplectic momentum scheme. In addition to the discrete equations of motion, further constraints, like initial conditions, final conditions and possible inequality path constraints can be imposed. To reduce the dimension of the constrained forced discrete Euler-Lagrange equations during the contact phase, we apply the discrete null space method with nodal reparametrisation, see [2]. Thereby, the minimal dimension of the mechanical system can be achieved by using the vector of incremental generalised coordinates u_{n+1} to reparametrise the configuration vector q_{n+1} in the neighbourhood of q_n . The nodal reparametrisation function $q_{n+1} = F_d(u_{n+1}, q_n)$ fulfills the kinematic (internal and external) constraints.

As it is illustrated in Figure 2, the optimal control problem considers a motion with a contact and a flight phase. The motion starts with the contact phase, where the foot is in contact with the ground. Using the contact null space matrix $P_S(q_n) \in \mathbb{R}^{36 \times 7}$, the constraint forces and the contact forces are eliminated from the discrete Euler-Lagrange equations. Furthermore, $g_S(q) = 0$ and $\lambda_{S_n}^3 < 0$ keep the foot fixed at the ground with a contact force in the correct direction (preventing interpenetration).



Figure 2: Time grid and dynamical constraints of the jumper's optimal control problem.

When the vertical component of the contact force vanishes $(\lambda_{S_n}^3 = 0)$, the jumper's foot contact to the ground is released and the flight phase starts, where the foot stays above the ground $(g_c(\mathbf{q}_n) > 0)$.



Figure 3: Snapshots of an optimised motion at the beginning, the contact release and at the end of the motion.

The constrained optimisation problem is formulated in terms of the discrete generalised coordinates u_d and the discrete actuation torques τ_d . The actuation of the monopedal jumper during the contact phase has an essential effect on the time of contact release and on the height and width of the jump. The optimal contact release time $t_{N_{\kappa}}$ is not known, therefore the note number N_{κ} is predefined and the time steps before and after the contact release are scaled by the parameters σ_1 , $\sigma_2 \in \mathbb{R}$. The scaling factors are part of the optimization variables. At the beginning of the manoeuvre, equality conditions guarantee an initial state $q_0(t_0) = q_0$, $p(t_0) = p_0$ of the jumper, whereby $p_0 = 0$ represents the zero conjugate momentum at the initial configuration, thus the motion starts at rest. During the optimal controlled motion several path constraints are present, e.g. an inequality constraint prevents the jumper's knee super-extension and another guarantees the correct orientation of the contact forces.

Figure 3 illustrates some configurations of an optimised motion, which starts at rest and the inequality constraints at the end ensure a minimum jump height of 1.5 m. The configuration at the beginning of the motion and the actuation forces are part of the optimisation problem. The goal is to investigate

high and long jumps with physiologically motivated cost functions and eventually a three-segmental foot model is applied (details can be found in [1]).

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Asynchronous variational Lie group integration for geometrically exact beam dynamics

François M.A. Demoures, François Gay-Balmaz, Thomas Leitz, Sigrid Leyendecker, Sina Ober-Blöbaum, Tudor S. Ratiu

The theory of discrete variational mechanics has its roots in the optimal control literature of the 1960's. The past ten years have seen a major development of discrete variational mechanics and corresponding numerical integrators, largely due to pioneering work by Jerrold Marsden and his collaborators, e.g. in [1]. The discrete Lagrangian L_d approximates the action in a time interval $[t_{j-1}, t_j]$. For linear vector spaces, i.e. $q \in \mathbb{R}^n$, this leads to the discrete action sum

$$S_d = \sum_N L_d\left(q_{j-1}, q_j\right)$$

and the discrete variational principle $\delta S_d = 0$ results in the discrete Euler-Lagrange equations

$$D_2 L_d \left(q_{j-1}, q_j \right) + D_1 L_d \left(q_j, q_{j+1} \right) = 0 \tag{1}$$

which due to the variational derivation are symplectic and conserve discrete momentum maps [1]. Asynchronous variational integrators (AVI), as described by Lew et. al. [2], offer the possibility to use different time steps for every element of the spatial discretization. The use of AVI promises less computational costs by increasing the time step sizes for slowly moving parts of the beam.

We review the kinematic description of the geometrically exact beam model in the ambient space \mathbb{R}^3 presented in [3]. The configuration of a beam is defined by specifying the position of its line of centroids by means of a map $\phi : [0, L] \to \mathbb{R}^3$, and the orientation of cross-sections at points $\phi(S)$ by means of a moving basis $\{d_1(S), d_2(S), d_3(S)\}$ attached to the cross section. The orientation of the moving basis is described with the help of an orthogonal transformation $\Lambda : [0, L] \to SO(3)$ such that

$$\boldsymbol{d}_I(S) = \Lambda(S)\boldsymbol{E}_I, \quad I = 1, 2, 3$$

where $\{E_1, E_2, E_3\}$ is a fixed basis referred to as the material frame. The configuration of the beam is thus completely determined by the maps ϕ and Λ in the configuration space

$$C^{\infty}\left([0,L],SE\left(3\right)\right)$$

with the Lie group SE(3) being the Euclidean group. If we take into account that the thickness of the rod is small compared to its length and that the material is homogenous and isotropic, we can

consider the stored energy to be given by a quadratic model. The discretization of the beam [0, L] in N elements $K \in \mathcal{T}$ (with two nodes in each K) and \mathcal{T} being the set of all elements, is done in a way that provides objective strain measures [4].

The mechanical system evolves on a Lie group. As a consequence, we use the discrete Lagrangian $\mathcal{L}_{K}^{j}: G \times G \to \mathbb{R}$, which can be defined by considering the contribution of the K-th element to the discrete reduced Lagrangian over the time interval $[t_{K}^{j}, t_{K}^{j+1}]$. Similar to the derivation of the discrete Euler-Lagrange equations on Lie groups developed by Bobenko, Suris [5] and Lee [6, 7], the discrete action sum becomes

$$\mathfrak{S}_d((\Lambda_d, x_d)) = \sum_{K \in \mathcal{T}} \sum_{1 \le j < N} \mathcal{L}_K^j$$

with (Λ_d, x_d) being the discrete curve in SE(3). In contrast to equation (1) for the linear vector space, applying the discrete Hamilton's principle to the discrete action sum leads to the discrete Euler-Lagrange equations for a node a

$$\begin{split} T_e^* L_{(F_a^{j-1}, H_a^{j-1})} \left(D_{F_a^{j-1}} \mathcal{L}_a^{j-1}, \quad D_{H_a^{j-1}} \mathcal{L}_a^{j-1} \right) \\ &- \operatorname{Ad}^*_{(F_a^j, H_a^j)^{-1}} T_e^* L_{(F_a^j, H_a^j)} \left(D_{F_a^j} \mathcal{L}_a^j, \quad D_{H_a^j} \mathcal{L}_a^j \right) \\ &+ T_e^* L_{(\Lambda_a^j, x_a^j)} \left(D_{\Lambda_a^j} \mathcal{L}_a^j, \quad D_{x_a^j} \mathcal{L}_a^j \right) = 0, \end{split}$$

for all $a \in \mathcal{N}$ with \mathcal{N} being the set of all nodes. Left multiplication by $g \in SE(3)$ is denoted by $L_g(f) = gf$ and $T^*L_g(f)$ is the contangent lifted action. For a node a, the discrete configuration g^j and the temporal configuration increment $f^j = (g^j)^{-1} g^{j+1}$ associated to this node are

$$g^{j} = (\Lambda_{a}^{j}, x_{a}^{j}) \text{ and } f^{j} = (F_{a}^{j}, H_{a}^{j}) := (\Lambda_{a}^{j}, x_{a}^{j})^{T} (\Lambda_{a}^{j+1}, x_{a}^{j+1}) = \left((\Lambda_{a}^{j})^{T} \Lambda_{a}^{j+1}, (\Lambda_{a}^{j})^{T} (x_{a}^{j+1} - x_{a}^{j}) \right)$$



Figure 1: Simulation of a geometrically exact beam with momentum initial conditions using AVI. Left: Snapshots of the motion. Right: Corresponding energy plot with no numerical dissipation.

The asynchronous Lie group variational integrator is implemented in Matlab using a priority queue described by [2] modified to allow time coincidences of two adjecent elements. This modification leads to a universal integrator, that allows synchronous as well as asynchronous time stepping. Figure 1 shows snapshots of the motion of a beam with nonzero initial conditions on momentum level and a prescribed deformed initial configuration. The corresponding energy plot shows that the presented integrator exhibits no numerical dissipation.

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Muscle paths in biomechanical multibody simulations

Ramona Maas, Sigrid Leyendecker

When simulating biomechanical motion with multibody systems representing bones and joints, the actuation of those systems can be implemented via Hill-type muscle models. The essential task of these models is to represent the typical force-length and force-velocity relation of real muscles. The Hill-model used in this work consists of a contractile component (CC) and a parallel linear elastic component (PEC), see Figure 1. The scalar force amount of the muscle force F_n^M in a time interval $[t_n, t_{n+1}]$ can be calculated via

$$F_n^M = (f_l)_n \cdot (f_v)_n \cdot A_n \cdot F_{max} + k_p \cdot (l_M)_n \tag{1}$$

assuming the parallel elasticity of the muscle to be proportional to the length of the muscle element l_M with the proportionality constant k_p . Herein, $f_l(l_M) \in [0, 1]$ is a factor related to the force length relation of the muscle, $f_v(v_M) \in [0, 1.4]$ represents the Hill-hyperbola like force-velocity relation,

This means, to calculate the actual muscle force, we need the actual muscle length in every time step as well as the force directions at the insertion points of the muscle. The muscle length and force direction is particularly related to the joint angle. Several studies use an alterable number of artificial 'via' points or 'wrapping' points to relate the muscle path to the joint angle, see for example [3]. The determination of such artificial points requires a lot of anatomical knowledge, which is not yet available for all biomechanical structures and the results are quite sensitive to the location of such points. In software packages like OpenSim, so-called 'muscle moment arms' are then calculated as partial derivatives of the muscle length around those artificial points with respect to the joint angle, see for example [5, 3]. $A \in [0,1]$ is the activity of the muscle and F_{max} is the maximal possible muscle force. The contraction velocity of the muscle is approximated via $v_{M,n} = \frac{(l_M)_{n+1} - (l_M)_n}{h}$ where the time step length is denoted by h. The muscle force $\boldsymbol{\tau}_n^m$ acting on the multibody system is then given by a multiplicative set up of the scalar force value and the force direction \boldsymbol{r}_n .

 $\boldsymbol{\tau}_n^m = F_n^M \cdot \boldsymbol{r}_n$



Figure 1: Hill-type muscle model

method	nodes	l_M	$oldsymbol{r}_n^1$			$oldsymbol{r}_n^2$	ı			elapsed time
optimisation	30	0.2148	0.6289	0.6954	-0.3478	0.	.2886	-0.9369	-0.1973	178.5s
optimisation	15	0.2145	0.6070	0.7123	-0.3524]	Ī0.	.2632	-0.9464	-0.1875	16.8s
optimisation	7	0.2130	0.5752	0.7751	-0.2617	0.	.2281	-0.9581	-0.1731	3.1s
semi-analytical	-	0.2146	[0.6192]	0.6800	-0.3326]	[0.	.2995	-0.9343	-0.1935]	1.7s

(2)

Table 1: Comparison of optimal muscle path using optimisation with different number of nodes and the semi-analytical path procedure combining lines, helices and orthodromes.

Our approach is to assume that the muscles and tendons are always under tension as it is described for example in [6], which means that tendons and muscles are supposed to follow the path of minimal distance between two insertion points. One possibility to find this path is to minimise the length of a tendon/muscle path between two insertion points over a joint, so that the path does not intersect the bodies \mathcal{K}_j . Let the path of a muscolotendon complex be discretised with $n_e + 1$ elements with the element length l_{e_i} . We get n_e nodes $\mathbf{E}_i \in \mathbb{R}^3$ between the insertion points $\mathbf{p}_1 \in \mathbb{R}^3$ and $\mathbf{p}_2 \in \mathbb{R}^3$, which are summarised in the optimisation variable $\mathbf{E} = [\mathbf{E}_1, \mathbf{E}_2, \dots, \mathbf{E}_{n_e}] \in \mathbb{R}^{3n_e}$. The constrained optimisation problem reads

$$\min_{E} l_{M} = \min_{E} \|p_{1} - E_{1}\| + \|E_{n_{e}} - p_{2}\| + \sum_{i=1}^{n_{e}-1} \|E_{i} - E_{i+1}\|$$
so that: $E_{i} \notin \mathcal{K}_{j}$
 $l_{e_{min}} < l_{e_{i}} < l_{e_{max}}$ for $i = 1, ..., n_{e} + 1$

$$(3)$$

Solving this problem yields a linear approximation of the length of the muscle path. The force direction at the insertion points is given via the normalised direction of the first and the last element. However, using an optimisation procedure like this during forward dynamics or optimal control simulations leads to several problems. First of all, the computation is very expensive, as for every muscle in every time step such an optimisation procedure has to be executed. Secondly, analytical gradients for optimal control simulations are difficult to calculate, since the muscle length is the solution of a parameter dependent optimisation.

Within our multibody simulation framework for biomechanical systems, we represent bones and joints via mostly smooth bodies like cylinders and spheres. It is known that the shortest path on the surface of a cylinder is a helix and the shortest path on a sphere is an orthodrome. We want to reduce computational effort in finding the path of minimal length around bodies and joints with an algorithm that determines this path as a G1-continuous combination of straight lines (wherever possible, i.e. whenever the straight line does not intersect the bodies or joints), helices and orthodromes. Note that G1 (geometrical) continuously joined curves share tangential directions, while the length of the tangent vectors might differ. Thus, the length of this path can directly be calculated as the sum of the length of the single parts and the force direction is given via the tangent vector at the insertion

points. From Table 1 it is obvious, that the simulation time can be significantly reduced by using this semi-analytical algorithm, whereas force direction and path length are comparable to those calculated with optimisations. In the left part of Figure 1, the muscle path resulting from an optimisation procedure with 30 nodes is depicted whereas the right part shows the semi-analytical solution, which is in this case a G1-continuous helix-line-helix combination.

Finally, when knowing the muscle path length, the scalar force amount and the force direction, the discrete forces can be calculated as described for example in [1]. Inserting them into the reduced dis-



Figure 2: Example of a muscle path. Left: solution of a 30 node optimisation. Right: the semianalytical solution.

crete version of the constrained Lagrange-d'Alembert principle, see for example [2, 1, 4], the resulting scheme can be solved either during forward dynamics simulations or it serves as constraints in optimal control simulations. Using the variational integrator resulting from the discrete Lagrange-d'Alembert principle instead of simply discretising the continuous equations of motion yields a structure preserving simulation. This means that the angular momentum is changing only and exactly according to the applied forces and the results show a good energy behaviour. The next step of this work is to implement this algorithm in forward dynamics and optimal control simulations of biomechanical systems like the human arm, finger and hand.

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A numerical approach to multiobjective optimal control of multibody dynamics

Maik Ringkamp, Sigrid Leyendecker, Sina Ober-Blöbaum

Recently, a couple of approaches have been developed that combine multiobjective optimization with direct discretization methods to approximate trajectories of optimal control problems (e.g. [1]) resulting in restricted optimization problems of high dimension. The solution set of a multiobjective optimization problem is called the Pareto set which consists of optimal compromise solutions. A common way to approximate the Pareto set is to start with at least one given Pareto solution and to evolve the Pareto set using continuation techniques. With our approach, we first approximate the feasible set of the multiobjective optimal control problem using a global root finding algorithm. Then, we choose appropriate points which provide initial guesses for the continuation of the Pareto set. After that, the continuation is performed by using a reference point method [2].

Multiobjective Optimal Control for Constrained Multibody Systems We apply the approach DMOCC [3] to a constrained formulation of multibody dynamics. That is a combination of discrete mechanics and optimal control (DMOC) [4] and a discrete null space method [5]. We consider a multibody system consisting of rigid bodies interconnected by joints. Each joint induces external constraints for the *n*-dimensional time dependent configuration vector $\mathbf{q}(t) \in Q$ in the configuration manifold Q. Moreover, the problem is given in a constrained formulation of rigid body dynamics such that internal constraints are required to fulfill the underlying kinematic assumptions. Altogether, a holonomic constraint function $\mathbf{g}: Q \to \mathbb{R}^m$ restricts \mathbf{q} to the constraint manifold $C := {\mathbf{q} \in Q | \mathbf{g}(\mathbf{q}) = \mathbf{0}}$. Under given regularity conditions, a local reparametrization function $\mathbf{F}: U \to C$ for an open subset $U \subseteq \mathbb{R}^{n-m}$ exists and can be given explicitly. Consequently, the configuration vector $\mathbf{q} \in Q$ and its velocities $\dot{\mathbf{q}} \in T_{\mathbf{q}(t)}Q$ in the tangent space are given by $\mathbf{q} = \mathbf{F}(\mathbf{u})$ and $\dot{\mathbf{q}} = D\mathbf{F}(\mathbf{u})\dot{\mathbf{u}}$. The aim is to minimize several objectives whereas the configuration vector has to satisfy the equations of motion given by the constrained Lagrange-d'Alembert principle in the time interval $[t_0, t_N]$ and boundary conditions. More detailed, an optimal control problem for constrained systems is defined as follows:

Problem 1

$$\min_{(\boldsymbol{u}, \dot{\boldsymbol{u}}, \boldsymbol{\tau}, t_N) \in [\boldsymbol{b}_l, \boldsymbol{b}_u]} J(\boldsymbol{u}, \dot{\boldsymbol{u}}, \boldsymbol{\tau}, t_N) = \int_{t_0}^{t_N} \boldsymbol{B}(\boldsymbol{u}(t), \dot{\boldsymbol{u}}(t), \boldsymbol{\tau}(t)) dt$$
(1)

s.t.
$$\boldsymbol{P}(\boldsymbol{F}(\boldsymbol{u}))^T \left[\partial_1 L(\boldsymbol{F}(\boldsymbol{u}), D\boldsymbol{F}(\boldsymbol{u})\dot{\boldsymbol{u}}) - \frac{d}{dt} \partial_2 L(\boldsymbol{F}(\boldsymbol{u}), D\boldsymbol{F}(\boldsymbol{u})\dot{\boldsymbol{u}}) + \boldsymbol{f}(\boldsymbol{u}, \dot{\boldsymbol{u}}, \boldsymbol{\tau}) \right] = \boldsymbol{0}$$
 (2)

with $\boldsymbol{B}: TU \oplus T^*U \to \mathbb{R}^k$ and $\boldsymbol{J}: TU \oplus T^*U \times]0, \infty[\to \mathbb{R}^k$ and as we consider multiobjective problems, we have k > 1 instead of k = 1 objectives. Here $\boldsymbol{f}: TU \oplus T^*U \to T^*Q$ is the force field and $\boldsymbol{P}: TU \to TC$ is the $n \times (n-m)$ nullspace matrix that spans the tangent space of the constraint manifold. Accordingly, $\boldsymbol{P}^T: T^*C \to T^*U$ maps the Euler-Lagrange equation (2) to the minimal dimensional space. \boldsymbol{b}_l and \boldsymbol{b}_u are lower and upper bounds on the optimization variables. Using a discretization of \boldsymbol{u} and $\boldsymbol{\tau}$ and finite differences for $\dot{\boldsymbol{u}}$ in the variational problem transforms Problem 1 in a finite dimensional restricted optimization problem with optimization variables $\boldsymbol{x} =$ $(\boldsymbol{u}^0, \dots, \boldsymbol{u}^N, \boldsymbol{\tau}^0, \dots, \boldsymbol{\tau}^{N-1}, t_N) \in \mathbb{R}^{(2N+1)(n-m)+1}$.

Multiobjective Optimization For a given multiobjective optimization problem with objective J and feasible set S (i.e. all points in $[b_l, b_u]$ that satisfy (2) and (3)), the following definitions clarify what is meant by the minimum:

- **Definition 1** (i) A vector $\mathbf{y} \in S$ is dominated by a vector $\mathbf{x} \in S$ (in short: $\mathbf{x} \prec \mathbf{y}$) with respect to Problem 1 if $\mathbf{J}(\mathbf{x}) \leq \mathbf{J}(\mathbf{y})$ and $\mathbf{J}(\mathbf{x}) \neq \mathbf{J}(\mathbf{y})$.
 - (ii) A vector $x \in S$ is called Pareto optimal or a Pareto point if there is no $y \in S$ which dominates x. Sometimes also its corresponding point in image space J(x) is called a Pareto point.
- (iii) The Pareto set \mathcal{P} is defined as the set of all Pareto points and the corresponding set in the image space $J(\mathcal{P})$ is called the Pareto front.

Minimizing a vector valued function means to find its Pareto set. To find the Pareto set of a multiobjective optimal control problem we propose the following solution strategy:

- 1. Computation of a rough approximation of the feasible set.
- 2. Sorting out dominated trajectories.
- 3. Evolution of the Pareto set using a continuation method starting from each remaining trajectory.

The first step can be done for example by using randomly chosen, possibly infeasible trajectories $x \in [b_l, b_u]$ as starting points for the minimization problem $\min_{x \in S} 1$. This leads to a finite set of trajectories $T \subseteq S$. After applying a test of dominance, we sort out all dominated trajectories and obtain the set $\mathcal{P}_T := \arg \min T$. We use a reference point optimization for the last step. This technique successively generates reference points $r \in \mathbb{R}^k$ such that no $x \in S$ exists with $J(x) \leq r$. Each of them is used for a distance minimization of $\min_{x \in S} ||J(x) - r||$. Thus, the vector valued objective J is transformed into a scalar valued auxiliary function such that standard minimization algorithms can be applied for the minimization.

Example: 4-Body Kinematic Chain The considered problem consists of four rigid bodies interconnected by two revolute joints and one spherical joint. The initial and final conditions (translation and rotation) are fully specified such that the kinematic chain moves from a straight to a closed position, performing a rest to rest maneuver. The objectives to minimize are the control effort J_1 for the complete maneuver and the required maneuver time J_2 . Figure 1 shows parts of the computed Pareto front and trajectories for the center of mass of each rigid body.



Figure 1: Left: Approximation of the computed Pareto front for the objectives J_1 and J_2 . Middle: Center of mass trajectories for the Pareto point ('o' in left). Right: Center of mass trajectory for the Pareto point ('+' in left).

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4 Activities

4.1 Teaching

Wintersemester 2012/2013

Dynamik starrer Körper (MB	, ME, WING, IP, BPT, CE)	
Vorlesung		S. Leyendecker
Übung + Tutorium		T. Gail, O.T. Kosmas
-		T. Leitz, M. Ringkamp
Mehrkörperdynamik (MB, MI	E, WING, TM, BPT, CE)	
Vorlesung		S. Leyendecker
Ubung		H. Lang
Theoretische Dynamik (MB 1	ME WING TM CE BPT)	
Vorlesung + Übung		H Lang
volicibility 0 build		II. Dang
Numerische Methoden in der	Mechanik (MB, ME, WING, TM, CE, BP	T)
$Vorlesung + \ddot{U}bung$		H. Lang
Dynamik nichtlinearer Balken	(MB. M. Ph. CE. ME. WING)	
Vorlesung		H. Lang
		8
Sommersemester 2012		
Statik und Festigkeitslehre (C	BI, ET, IP, LSE, ME, MT, WING, WW)	
Vorlesung		S. Leyendecker
$\ddot{\text{U}}$ bung + Tutorium		T. Leitz, O.T. Kosmas
geprüft	405	
Diamachanil (MT)		
Vorlagung – Übung		II Lana
voriesung + Obung	70	п. Lang
gepruit	12	
Geometrische Mechanik und o	eometrische Integratoren	

Geometrische Mechanik und geometrische Integratoren (MB, ME, CE, BPT, WING, M, TM, Ph) Vorlesung + Übung S. Leyendecker 3 geprüft Numerische Methoden in der Mechanik (IP, MP, ME, WING, M, TM, Ph) Vorlesung + ÜbungH. Lang 17geprüft Theoretische Dynamik II (M, TM, MB, ME, CE, BPT, WING, Ph) $Vorlesung + \ddot{U}bung$ H. Lang 0 geprüft Rechnerunterstützte Produktentwicklung (RPE) Versuch 6: Mehrkörpersimulation in Simulink (MB, ME, WING) Praktikum M. Koch, O.T. Kosmas T. Leitz, R. Maas Teilnehmer 70

Wintersemester 2011/2012

Dynamik starrer Körp	er (MB, ME, WING, IP, BPT, CE)	
Vorlesung		S. Leyendecker
Übung + Tutoriu	ım	H. Lang, T. Leitz
		O.T. Kosmas
geprüft	591+169 (SS 2012)	
Mehrkörperdynamik (MB, ME, WING, BPT, CE)	
Vorlesung		S. Leyendecker
Übung		H. Lang
gepr üft	13+3 (SS 2012)	
Theoretische Dynamik	I (M, TM, MB, ME, CE, BPT, WING, Ph)	
Vorlesung + \ddot{U} bu	ing	H. Lang
gepr üft	2	

Sommersemester 2011

Statik und Festigkeitslehre (O	CBI, ET, IP, LSE, ME, MT, WING, WW)	
Vorlesung		S. Leyendecker
$\ddot{\mathrm{U}}\mathrm{bung} + \mathrm{Tutorium}$		V. Barth, O.T. Kosmas
gepr üft	475	
Biomechanik (MT)		
$Vorlesung + \ddot{U}bung$		S. Leyendecker
geprüft	$31+11 (WS \ 11/12)$	



4.2 Theses

Diploma theses

- Tobias Gail Computing time investigations for variational multirate schemes
- Alexander Werner (supervision with DLR) Optimization-based generation of optimal walking trajectories for biped robots

Master theses

• Thomas Pircher Biomechanical model of the muscle tendon network in human fingers

Bachelor theses

- Johannes Rudolph (supervision with Siemens AG) Modellierung und Simulation des Unwuchtverhaltens eines CT-Systems
- Marion Stadler Index investigations in discrete mechanics and optimal control for differential algebraic systems

4.3 Seminar for Mechanics

together with the Chair of Applied Mechanics LTM

04.04.2011	Jean-Paul Pelteret CERECAM, University of Cape Town, South Africa Computational Model of Tissue in the Human Upper Airway
05.04.2011	Louis Komzsik Chief Numerical Analyst of Siemens Industry Division, PLMS in California, USA Introduction to industrial rotor dynamics
19.04.2011	Vera Luchscheider Chair of Applied Mechanics, FAU Erlangen-Nuremberg, Germany Bruchmechanische Ermittlung der Ausfallwahrscheinlichkeit von Wälzlagerbauteilen mit Einschlüssen
20.06.2011	Matjaž Hriberšek University of Maribor, Slovenia Numerical modeling of dilute suspension flows of magnetic particles by the Subdomain Boundary Element Method
28.06.2011	Holger Lang ITWM Kaiserslautern, Germany Geometrisch exakte Cosseratsche Balken für die Mehrkörpersimulation
29.06.2011	Thorsten Schindler INRIA Grenoble, France Nichtglatte MKS in industrieller Anwendung und theoretischer Analyse
05.07.2011	Jürgen Metzger TRW Automotive, Alfdorf, Germany Characterization and Evaluation of Frontal Crash Pulses for USNCAP 2011

06.07.2011	Bülent Yagimli UniBW Munich, Germany Experimentelle Untersuchungen und Erstellung eines Materialmodells zur Beschreibung von Aushärtevorgängen
12.07.2011	Holger Böse ISC Würzburg, Germany Smart Materials zur gezielten Beeinflussung mechanischer Systeme
05.10.2011	Indresan Govender University of Cape Town, South Africa Flow modeling in tumbling mills
19.01.2012	Philipp Landkammer Ingenieurbüro KAE GmbH, Hausen b. Forchheim, Germany Das Antwortspektrenverfahren für Erdbebensimulationen
26.01.2012	Fernando Jiménez Alburquerque Instituto de Ciencias Matemàticas ICMAT-CSIC, Madrid, Spain On Discrete Mechanics for Optimal Control Theory
07.02.2012	Daniel Riedlbauer Chair of Applied Mechanics, FAU Erlangen-Nuremberg, Germany Thermomechanical Modelling & Simulation of Electron Beam Melting
06.03.2012	Francesco dell'Isola DISG, Università di Roma "La Sapienza", Rome, Italy How contact interactions may depend on the shape of Cauchy cuts in N-th gradient continua: approach "à la D' Alembert"
23.03.2012	Ellen Kuhl Computational Biomechanics Laboratory, Stanford University, USA Computational Optogenetics: A Novel Continuum Framework for the Photoelectro- chemistry of Living Systems
03.04.2012	Kim-Henning Sauerland Lehrstuhl für Technische Mechanik, University of Paderborn, Germany Process Simulation and Two Scale Tool Simulation related to Hybrid Forming
09.05.2012	Olivier Verdier Department of Mathematical Sciences, NTNU Trondheim, Norwegen Geometric Generalisations of the Shake and Rattle methods
21.05.2012	Oleg M. Zarechnyy Department of Aerospace Engineering, Iowa State University, Ames, IA, USA Modeling and Simulation of Strain-Induced Phase Transformations in Rotational Dia- mond Anvil Cell

22.05.2012	Zoufine Bare Fraunhofer Institut für Techno- und Wirtschaftsmathematik, Kaiserslautern, Germany Asymptotic dimension reduction for linearized contact of thin fibers and simulation of textiles based on 1D models including large deformation
05.06.2012	Barbara Röhrnbauer, Edoardo Mazza Institute of Mechanical Systems, Swiss Federal Institute of Technology Zurich, Switzer- land Mechanical characterization and modeling of prosthetic meshes at different length scales
12.06.2012	Prashant Saxena Department of Mathematics, University of Glasgow, UK Nonlinear magneto-elasticity: some boundary value problems
26.06.2012	Valery Levitas Department of Aerospace Engineering, Iowa State University, Ames, IA, USA Stress- and Surface-induced Phase Transformations: Phase Field Approach
03.07.2012	Karali Patra Mechanical Engineering, Indian Institute of Technology Patna, India Study on mechanical and dielectric behavior of VHB 4910 for sensors and actuators applications
09.08.2012	Wencheng Li Northwestern Polytechnic University, China Introduction of My Research Interesting on Structure Preserving Methods
06.09.2012	Kathrin Flaßkamp Deparment of Mathematics, University of Paderborn, Germany Variational Formulation and Optimal Control of Hybrid Lagrangian Systems
21.09.2012	Roger Bustamante Departamento de Ingenieria Mecànica, Universidad de Chile, Chile Implicit constitutive relations for electro-elastic bodies
27.09.2012	Joachim Linn Fraunhofer-Institut für Techno- und Wirtschaftsmathematik, Kaiserslautern, Germany Viscoelastic Cosserat rods of KelvinVoigt and generalized Maxwell type
22.10.2012	Markus Lazar Heisenberg Research Group, Continuum Mechanics, Department of Physics, Darmstadt University of Technology, Germany Non-singular Dislocations in the Theory of Gradient Elaticity
05.11.2012	Hossein Talebi Institute of Structure Mechanics, Bauhaus-University Weimar, Germany Single and Multi Scale Methods for Modeling Fracture and Crack Propagation: Methods, Software and Tools

19.11.2012	Frank Fischer Structure Research Lab, Beiersdorf AG, Hamburg, Germany The detailed structure of human skin layers
20.11.2012	Tobias Gail Chair of Applied Dynamics, FAU Erlangen-Nuremberg, Germany Computing time investigations of variational multirate schemes
22.11.2012	Axel Kohlmeyer Abdus Salam International Centre for Theoretical Physics (ICTP), Trieste, Italy und Institute for Computational Molecular Simulations (ICMS), Temple University, Philadelphia, USA Accelerating classical MD for multi-core CPUs and GPUs
04.12.2012	Maik Ringkamp Chair of Applied Dynamics, FAU Erlangen-Nuremberg, Germany Using Discrete Mechanics and Reference Point Techniques to Solve Multiobjective Op- timal Control Problems in Space Mission Design and Optimal Control Multi-Body Dy- namics
05.12.2012	Johannes Rudolph Chair of Applied Dynamics, FAU Erlangen-Nuremberg, Germany Modellierung und Simulation des Unwuchtverhaltens eines CT-Systems
05.12.2012	Marion Stadler Chair of Applied Dynamics, FAU Erlangen-Nuremberg, Germany Index Investigations in Discrete Mechanics and Optimal Control for Differential Alge- braic Systems
06.12.2012	François M.A. Demoures EPFL/ENAC/IIC/IBOIS (Laboratoire de construction en bois, Lausanne, Switzerland)

Lie group and Lie algebra variational integrators for flexible beam and plate in \mathbb{R}^3





4.4 Press releases

Nürnberger Nachrichten, Tuesday, 6 March 2012



Nürnberger Nachrichten, Saturday, 20 October 2012

Uni Kurier Magazin, Nr. 112, September 2012



Prof. Dr. Sigrid Leyendecker

Sigrid Leyendecker ist seit April 2011 Inhaberin des Lehrstuhls für Technische Dynamik.

Ihrem Studium der Technomathematik an der TU Kaiserslautern (Diplom 2002) folgte 2006 die Promotion am dortigen Lehr-

stuhl für Technische Mechanik zum Thema Simulation flexibler Mehrkörperdynamik. Mit einem Feodor Lynen-Forschungsstipendium der Alexander von Humboldt-Stiftung verbrachte sie anschließend zwei Jahre am California Institute of Technology. Dort arbeitete sie an der Entwicklung numerischer Methoden für Optimalsteuerungsprobleme. Im Herbst 2008 ging sie als Gastdozentin an die FU Berlin, bevor sie im Mai 2009 als Leiterin der Nachwuchsgruppe "Simulation und optimale Steuerung der Dynamik von Mehrkörpersystemen in der Biomechanik und Robotik" im Emmy Noether-Programm der Deutschen Forschungsgemeinschaft (DFG) wieder an die TU Kaiserslautern zurückkehrte (Habilitation 2011).

Forschungsschwerpunkte liegen in der Entwicklung effizienter numerischer Verfahren zur strukturerhaltenden Simulation und Optimalsteuerung dynamischer Systeme. Dazu gehören etwa industrielle und medizinische Roboter wie auch Bewegungsvorgänge des menschlichen Körpers in der Biomechanik oder die Dynamik von Polymerketten. Neben der Numerik spielt die Modellierung der nichtlinearen Systeme eine entscheidende Rolle.

5 Publications

5.1 Reviewed journal publications

- H. Lang, J. Linn, and M. Arnold. Multibody dynamics simulation of geometrically exact Cosserat rods. Multibody Dynamics, Vol. 25(3), pp. 285-312, 2011.
- R. Maas, T. Siebert, and S. Leyendecker. On the relevance of structure preservation to simulations of muscle actuated movements. Biomech. Model. Mechanobiol., DOI 10.1007/s10237-011-0332-0, Vol. 11, pp. 543-556, 2012.
- 3. S. Leyendecker, C. Hartmann, and M.W. Koch. Variational collision integrator for polymer chains. J. Comput. Phys., DOI 10.1016/j.jcp.2012.01.017, Vol. 231, pp. 3896-3911, 2012.
- G. Johnson, M. Ortiz, and S. Leyendecker. A linear programming-based algorithm for the signed separation of (non-smooth) convex bodies. Comput. Methods Appl. Mech. Engrg., DOI 10.1016/j.cma.2012.04.006, Vol. 232-236, pp. 49-67, 2012.
- 5. M.W. Koch, and S. Leyendecker. Energy momentum consistent force formulation for the optimal control of multibody systems. Multibody Syst. Dyn., DOI 10.1007/s11044-012-9332-9, 2012.
- 6. O.T. Kosmas, and D.S. Vlachos. Local path fitting: a new approach to variational integrators. Journal of Computational and Applied Mathematics, Vol. 236(10), pp. 2632-2642, 2012.
- 7. O.T. Kosmas, and D.S. Vlachos. Simulated annealing for optimal ship routing. Computers & Operations Research, Vol. 39(3), pp. 576-581, 2012.
- H. Lang, and M. Arnold. Numerical aspects in the dynamic simulation of geometrically exact rods. Applied Numerical Mathematics, DOI 10.1016/j.apnum.2012.06.011, Vol. 62(10), pp. 1411-1427, 2012.
- M. Ringkamp, S. Ober-Blöbaum, M. Dellnitz, and O. Schütze. Handling High Dimensional Problems with Multi-Objective Continuation Methods via Successive Approximation of the Tangent Space. Engineering Optimization, DOI:10.1080/0305215X.2011.634407, Vol. 44(9), pp. 1117-1146, 2012.

5.2 Reviewed proceeding publications

- M.W. Koch, and S. Leyendecker. Structure preserving simulation of compass gait and monopedal jumping. In Proceedings of the Multibody Dynamics, ECCOMAS Thematic Conference, USB, Brussels, Belgium, 4-7 July 2011.
- 2. S. Leyendecker, and S. Ober-Blöbaum. A variational approach to multirate integration for constrained systems. In Proceedings of the Multibody Dynamics, ECCOMAS Thematic Conference, USB, Brussels, Belgium, 4-7 July 2011.
- 3. M.W. Koch, and S. Leyendecker. *Optimal control of multibody dynamics with contact.* In Proc. Appl. Math. Mech., PAMM, Vol. 11, pp. 51-53, 18-21 April 2011.
- 4. S. Leyendecker, and S. Ober-Blöbaum. Variational multirate integration of constrained dynamics. In Proc. Appl. Math. Mech., PAMM, Vol. 11, pp. 53-54, 18-21 April 2011.
- 5. R. Maas, T. Siebert, and S. Leyendecker. Structure preserving simulation of muscle actuated movements. In Proc. Appl. Math. Mech., PAMM, Vol. 11, pp. 101-102, 18-21 April 2011.

- 6. O.T. Kosmas. Charged particle in an electromagnetic field using variational integrators. In AIP Conference Proceedings of International Conference of Numerical Analysis and Applied Mathematics, ICNAAM, Vol. 1389, pp. 1927-1931, 19-25 September 2011.
- M.W. Koch, and S. Leyendecker. Structure preserving simulation of monopedal jumping. In Proc. Appl. Math. Mech., PAMM, Vol. 12(1), pp. 71-72, 26-30 March 2012.
- O.T. Kosmas, and S. Leyendecker. Phase lag analysis of variational integrators using interpolation techniques. In Proc. Appl. Math. Mech., PAMM, Vol. 12(1), pp. 677-678, 26-30 March 2012.
- 9. S. Leyendecker, G. Johnson, and M. Ortiz. *Planned contacts and collision avoidance on optimal control problems*. In Proc. Appl. Math. Mech., PAMM, Vol. 12(1), pp. 77-78, 26-30 March 2012.
- R. Maas, and S. Leyendecker. Optimal control simulations of human arm motion. In Proc. Appl. Math. Mech., PAMM, Vol. 12(1), pp. 99-100, 26-30 March 2012.
- S. Leyendecker, C. Hartmann, M.W. Koch, G. Johnson, and M. Ortiz. Variational collision integrators in forward dynamics and optimal control. In Proceedings of the Seventh International Conference of the Croatian Society of Mechanics, ICCSM, 17 pages, Zadar, Croatia, 22-25 May 2012.
- M.W. Koch, and S. Leyendecker. Structure preserving simulation of monopedal jumping. In Proceedings of the Second Joint International Conference on Multibody System Dynamics, IMSD, Stuttgart, Germany, 29 May - 1 June 2012.
- H. Lang, and J. Linn. On the effect of the discretisation scheme on the eigenfrequencies and modes of shear flexible rods. In Proceedings of the Second Joint International Conference on Multibody System Dynamics, IMSD, Stuttgart, Germany, 29 May - 1 June, 2012.
- S. Leyendecker, G. Johnson, and M. Ortiz. *Planned contacts and collision avoidance in optimal control problems*. In Proceedings of the Second Joint International Conference on Multibody System Dynamics, IMSD, Stuttgart, Germany, 29 May 1 June 2012.
- J. Linn, H. Lang, and A. Tuganov. Geometrically exact Cosserat rods with Kelvin-Voigt type viscous damping. In Proceedings of the Second Joint International Conference on Multibody System Dynamics, IMSD, Stuttgart, Germany, 29 May - 1 June, 2012.
- R. Maas, and S. Leyendecker. Optimal control of biomechanical motion using physiologically motivated cost functions. In Proceedings of the Second Joint International Conference on Multibody System Dynamics, IMSD, Stuttgart, Germany, 29 May - 1 June 2012.
- M. Schulze, S. Dietz, A. Tuganov, H. Lang, and J. Linn. Integration of nonlinear models of flexible body deformation in multibody system dynamics. In Proceedings of the Second Joint International Conference on Multibody System Dynamics, IMSD, Stuttgart, Germany, 29 May - 1 June, 2012.
- 18. M. Ringkamp, A. Walther, P. Reinold, K. Witting, M. Dellnitz, and A. Traechtler. Using algorithmic differentiation for the multiobjective optimization of a test vehicle. In Proceedings of EVOLVE International Conference, Mexico City, Mexico, 7-9 August 2012.
- O.T. Kosmas, and S. Leyendecker. Phase fitted variational integrators using interpolation techniques on non regular grids. In AIP Conference Proceedings of International Conference of Numerical Analysis and Applied Mathematics, ICNAAM, Vol. 1479, pp. 2402-2406, Kos, Greece, 19-22 September 2012.

- S. Reitelshöfer, M. Landgraf, J. Franke, and S. Leyendecker. Qualifizierung Dielektrischer-Elastomer-Aktoren zum Einsatz als künstliche Muskeln in hochdynamischen N-DOF Roboterkinematiken. In Tagungsband des 6. Bionik-Kongress, Bremen, Germany, 26-27 October 2012.
- 21. S. Ober-Blöbaum, M. Ringkamp, and G. zum Felde. Solving Multiobjective Optimal Control Problems in Space Mission Design using Discrete Mechanics and Reference Point Techniques. In Proceedings of the 51-th IEEE Conference on Decision and Control, Maui, HI, USA, 10-13 December 2012.

5.3 Talks

- 1. M.W. Koch, and S. Leyendecker. *Optimal control of multibody dynamics with contact.* GAMM Annual Meeting, Graz, Austria, 18-21 April 2011.
- 2. S. Leyendecker, and S. Ober-Blöbaum. Variational multirate integration of constrained dynamics. GAMM Annual Meeting, Graz, Austria, 18-21 April 2011.
- 3. R. Maas, T. Siebert, and S. Leyendecker. Structure preserving simulation of muscle actuated movements. GAMM Annual Meeting, Graz, Austria, 18-21 April 2011.
- 4. S. Leyendecker, and S. Ober-Blöbaum. Variational integration of constrained dynamics on different time scales. Poster (won the Simtech poster award), International Conference on Simulation Technology, Stuttgart, Germany, 14-17 June 2011.
- 5. M.W. Koch, and S. Leyendecker. Structure preserving simulation of compass gait and monopedal jumping. ECCOMAS Thematic Conference, Brussels, Belgium, 4-7 July 2011.
- 6. S. Leyendecker, and S. Ober-Blöbaum. A variational approach to multirate integration for constrained systems. ECCOMAS Thematic Conference, Brussels, Belgium, 4-7 July 2011.
- S. Leyendecker. Simulationsmethoden f
 ür Optimalsteuerungsprobleme in der Mechanik. Antrittsvorlesung, Department of Mechanical Engineering, University of Erlangen-Nuremberg, Erlangen, Germany, 22 July 2011.
- 8. S. Leyendecker, and S. Ober-Blöbaum. A variational approach to multirate integration for constrained systems. Applied Dynamics and Geometric Mechanics workshop, Mathematisches Forschungsinstitut Oberwolfach, Oberwolfach, Germany, 14-20 August 2011.
- 9. O.T. Kosmas. Charged particle in an electromagnetic field using variational integrators. International Conference of Numerical Analysis and Applied Mathematics, ICNAAM, Halkidiki, Greece, 19-25 September 2011.
- 10. S. Leyendecker. *Optimisation and optimal control of multibody dynamics*. Invited lecture, Multibody System Dynamics, Robotics and Control Workshop, Linz, Austria, 26-27 September 2011.
- S. Leyendecker. Structure preserving simulation methods for constrained dynamical systems and their optimal control. Discrete Mechanics and Integrators Workshop, Lausanne, Switzerland, 6 October 2011.
- 12. S. Leyendecker, M.W. Koch, and R. Maas. *Optimisation and optimal control of multibody dynamics*. Recent Trends in Optimisation for Computational Solid Mechanics, EUROMECH Colloquium 522, Erlangen, Germany, 10-13 October 2011.

- 13. M.W. Koch, and S. Leyendecker. *Structure preserving simulation of monopedal jumping.* GAMM Annual Meeting, Darmstadt, Germany, 26-30 March 2012.
- 14. O.T. Kosmas, and S. Leyendecker. *Phase lag analysis of variational integrators using interpolation techniques.* GAMM Annual Meeting, Darmstadt, Germany, 26-30 March 2012.
- 15. H. Lang. A discrete Cosserat rod model taking into account the effect of torsion warping suitable for the dynamik simulation of wind turbine rotor blades. GAMM Annual Meeting, Darmstadt, Germany, 26-30 March 2012.
- 16. T. Leitz, and K. Willner. Simulation of the elastohydrodynamic contact with a piezo-viscous fluid. GAMM Annual Meeting, Darmstadt, Germany, 26-30 March 2012.
- 17. S. Leyendecker, G. Johnson, and M. Ortiz. *Planned contacts and collision avoidance on optimal control problems*. GAMM Annual Meeting, Darmstadt, Germany, 26-30 March 2012.
- 18. R. Maas, and S. Leyendecker. *Optimal control simulations of human arm motion*. GAMM Annual Meeting, Darmstadt, Germany, 26-30 March 2012.
- 19. S. Ober-Blöbaum, and S. Leyendecker. Construction and analysis of variational multirate integrators. GAMM Annual Meeting, Darmstadt, Germany, 26-30 March 2012.
- 20. S. Leyendecker. Variational collision integrators in forward dynamics and optimal control. Invited plenary lecture, the Seventh International Conference of the Croatian Society of Mechanics, ICCSM, Zadar, Croatia, 22-25 May 2012.
- M.W. Koch, and S. Leyendecker. Structure preserving simulation of monopedal jumping. The Second Joint International Conference on Multibody System Dynamics, IMSD, Stuttgart, Germany, 29 May - 1 June 2012.
- 22. H. Lang, and J. Linn. On the effect of the discretisation scheme on the eigenfrequencies and modes of shear flexible rods. The Second Joint International Conference on Multibody System Dynamics IMSD, Stuttgart, Germany, 29 May - 1 June, 2012.
- S. Leyendecker, G. Johnson, and M. Ortiz. *Planned contacts and collision avoidance in optimal control problems*. The Second Joint International Conference on Multibody System Dynamics, IMSD, Stuttgart, Germany, 29 May 1 June 2012.
- 24. J. Linn, H. Lang, and A. Tuganov. Geometrically exact Cosserat rods with Kelvin-Voigt type viscous damping. The Second Joint International Conference on Multibody System Dynamics IMSD, Stuttgart, Germany, 29 May - 1 June, 2012.
- 25. R. Maas, and S. Leyendecker. Optimal control of biomechanical motion using physiologically motivated cost functions. The Second Joint International Conference on Multibody System Dynamics, IMSD, Stuttgart, Germany, 29 May - 1 June 2012.
- 26. M. Schulze, S. Dietz, A. Tuganov, H. Lang, and J. Linn. IIntegration of nonlinear models of flexible body deformation in multibody system dynamics. The Second Joint International Conference on Multibody System Dynamics, IMSD, Stuttgart, Germany, 29 May 1 June, 2012.
- S. Leyendecker. Variational collision integrators in forward dynamics and optimal control. Invited lecture, Chair of Mechanics and Robotics, University of Duisburg-Essen, Duisburg, Germany, 13 June 2012.

- 28. M. Ringkamp, A. Walther, P. Reinold, K. Witting, M. Dellnitz, and A. Traechtler. Using Algorithmic Differentiation for the Multiobjective Optimization of a Test Vehicle. The EVOLVE International Conference, Mexico City, Mexico, 7-9 August 2012.
- 29. O.T. Kosmas, and S. Leyendecker. *Phase fitted variational integrators using interpolation techniques on non regular grids*. International Conference of Numerical Analysis and Applied Mathematics, ICNAAM, Kos, Greece, 19-22 September 2012.
- S. Reitelshöfer, M. Landgraf, J. Franke, and S. Leyendecker. Qualifizierung Dielektrischer-Elastomer-Aktoren zum Einsatz als künstliche Muskeln in hochdynamischen N-DOF Roboterkinematiken. 6. Bionik-Kongress, Bremen, Germany, 26-27 October 2012.
- 31. S. Ober-Blöbaum, M. Ringkamp, and G. zum Felde. Solving Multiobjective Optimal Control Problems in Space Mission Design using Discrete Mechanics and Reference Point Techniques. The 51-th IEEE Conference on Decision and Control, Maui, HI, USA, 10-13 December 2012.

6 Social events

Birthday parties



Christmas party 2011 together with LTM



Department summer party 2012



Barbecue party summer 2012



Christmas party 2012 together with LTM

