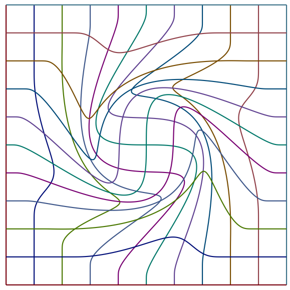
Dr. Rodrigo T. Sato Martín de Almagro

Geometric beam theory

In this course we aim to provide an overview of the theory of continuum mechanics in general and of beams in particular from a geometric and variational point of view. There will be a heavy emphasis on the theoretical and mathematical aspects.

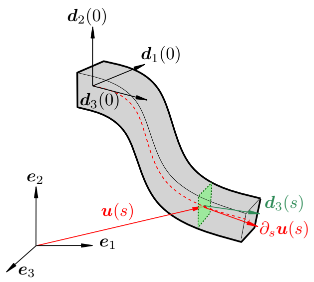
We will begin by learning the mathematical tools necessary to study the subject in depth. However, only a good command of linear algebra is assumed. A refresher on vector spaces and tensors will be given and then, we will move on to smooth manifolds.

Armed with those tools, we will dive into the subject of continuum mechanics, from the general nonlinear setting to the linear setting the students might already be familiar with. We will see how the resulting equations of motion can also be obtained as the Euler-Lagrange equations of a variational principle.



A diffeomorphism modelling the nonlinear deformation of a continuum.

We will derive the equations for a beam from the linear theory, recuperating the classical Euler-Bernuolli and Timoshenko beams.



Sketch of a geometrically exact beam model.

Finally, we will introduce and study a non-linear beam model known as the geometrically exact beam and see how it may be derived from the general nonlinear theory of continuum mechanics. The study of this model leads us to the introduction of Lie groups. These will tie with our initial study of smooth manifolds.

If time permits, we will also see how Lie groups can help us derive conservation laws from variational principles via Noether's theorem and how this in turn allows us to derive the balance laws from continuum mechanics.

Addressed audience: Master's students in engineering, mathematics and physics

Recommended previous knowledge: Knowledge of dynamics and statics, elastostatics, linear algebra and differential equations.

Date and location: Wednesday and Friday from 10:15 to 11:45. The lecture will take place online in livestream via Zoom.

References:

R. Abraham and J. E. Marsden. Foundations of mechanics.

Javier Bonet and Richard D. Wood. Nonlinear continuum mechanics for finite element analysis.

Philippe G. Ciarlet. Mathematical elasticity. Studies in Mathematics and its Applications. Three-dimensional elasticity.

H. Goldstein, C.P. Poole, and J.L. Safko. Classical Mechanics.

D. D. Holm. Geometric mechanics. Part II. Rotating, translating and rolling.

J. M. Lee. Introduction to Smooth Manifolds.

Julia Mergheim. Lecture notes - Nonlinear Finite Element Methods. July 2011.

Jerrold E. Marsden and Thomas J. R. Hughes. Mathematical foundations of elasticity

Peter J. Olver. Applications of Lie groups to differential equations.

H.-R. Schwarz. Finite element methods.

J. Simo. A finite strain beam formulation. The three-dimensional dynamic problem. Part I.