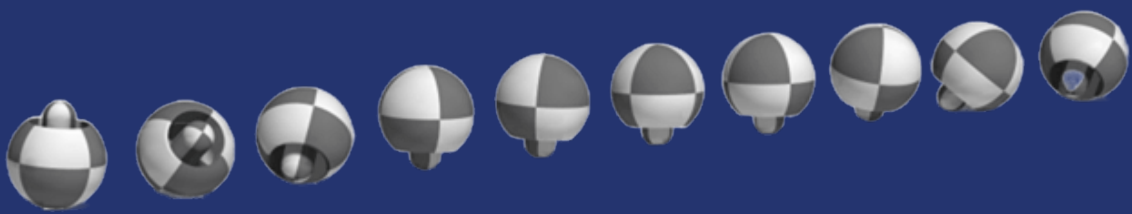
ENNSD 2025



12th Annual Symposium of the European Network for Nonsmooth Dynamics

Erlangen, September 30th – October 2nd, 2025

Friedrich-Alexander-Universität
Technische Fakultät
Institute of Applied Dynamics

Contents

About

Welcome

Welcome to the European Network for Nonsmooth Dynamics (ENNSD) Symposium! We are excited to bring together researchers and practitioners from around the world to share their insights and latest findings in the field of nonsmooth dynamics. This year's symposium promises to be an engaging and enriching experience, with a diverse program of talks, discussions, and networking opportunities.
For the 12th ENNSD Symposium we will have different types of sessions to foster interaction and discussion among participants. The different formats are:

- *Didactic Introduction*: The session begins with a 15-minute didactic introduction to the research field/topic, followed by the main presentation (30 minutes) and concludes with 15 minutes for questions and discussion.
- *Moderated Open Discussion*: After a 30-minute presentation, the speaker introduces a question or set of questions with a few slides, opening a 30-minute moderated discussion.
- *PhD talk*: The presentation is given by a PhD student and lasts 20 minutes, followed by 10 minutes for questions and discussion.
- *Round Table Discussion*: Discussion of a specific topic, chaired by a moderator. The session starts with a brief introduction (5-10 minutes) to the topic, followed by a 60-minute open discussion among the participants.

European Network for Nonsmooth Dynamics (ENNSD)

The ENNSD provides a dynamic platform for collaboration through symposia, summer schools, and workshops dedicated to the study of nonsmooth dynamics. Organized annually, the symposium has established itself in various countries such as Switzerland, France, Belgium, the Netherlands, and Germany. The focus of ENNSD is on the nonlinear dynamics of nonsmooth mechanical systems, leveraging concepts from convex analysis and set-valued analysis, inspired by the work of Jean Jacques Moreau. Topics of interest include, but are not limited to:

- Modeling of mechanical systems with frictional unilateral constraints
- Formalisms for describing nonsmooth dynamical systems, e.g., variational inequalities, (measure) differential inclusions
- Numerical methods for the simulation of nonsmooth dynamical systems
- Nonlinear dynamics and control of nonsmooth dynamical systems

Since its inception in 2012, the ENNSD Symposium has aimed to foster an enriching exchange among experts and encourage collaborative learning. The symposium diverges from traditional conferences by offering speakers extended time slots, encouraging thorough discussion and interaction.

Organizing committee

Local organizer:	Giuseppe Capobianco, FAU Erlangen-Nürnberg, Germany
Steering committee:	Vincent Acary, INRIA Grenoble - Rhône-Alpes, France Olivier Brüls, University of Liège, Belgium Remco Leine, University of Stuttgart, Germany Simon Eugster, Eindhoven University of Technology, Netherlands

Venue

The 12th ENNSD Symposium will be held at the Institute for Applied Dynamics of the FAU Erlangen-Nürnberg, Germany.

Institute for Applied Dynamics (LTD)
Room H17 (ground floor)
Immerwahrstraße 1
91058 Erlangen
Germany

How to get there from the city center:
The venue is easily accessible by bus. The closest bus stops are "Erlangen Süd" or "Erlangen Stettiner Str". You can also easily reach the venue by bike (10-15 min ride). Moreover, there are several parking options available nearby.

Timetable

Tuesday, September 30th

09:00–09:30	Registration
09:30–09:45	Welcome remarks
09:45–10:45	Nonsmooth Dynamics: A Journey from Convex Analysis to Computer Code ***Remco Leine*** *Chair: Olivier Brüls*
10:45–11:30	Coffee Break
11:30–12:30	A non-non-smooth model for wheel-rail contact ***Martin Arnold*** *Chair: Simon Eugster*
12:30–14:00	Lunch
14:00–15:00	On the accelerated solution of stiff nonsmooth mechanical systems ***Jonas Breuling*** *Chair: Vincent Acary*
15:00–15:30	Coffee Break
15:30-16:30	Event-driven Dynamics of Soft-sphere Systems: Bridging Newtonian Mechanics and Non-Smooth Dynamics ***Thorsten Pöschel*** *Chair: Remco Leine*
17:30–19:00	Guided Tour: Botanical Garden Erlangen

For the guided tour, please meet at the entrance of the greenhouse of the Botanical Garden (Loschgestraße 1, 91054 Erlangen) at 17:25. You can take the bus number 286 from the bus stop “Erlangen Süd” (close to the venue) to the bus stop “Hugenottenplatz” and then walk from there.

Wednesday, October 1st

09:00–10:00	Semi-resolved FEM-PFEM-DEM models for complex immersed granular free surfaces flows ***Vincent Legat*** *Chair: Olivier Brüls*
10:00–10:30	A Non-Smooth Model for the Lateral Dynamics of Drill Bits ***Kaidong Chen*** *Chair: Remco Leine*
10:30–11:15	Coffee Break
11:15–12:30	The ENNSD community ***Round Table Discussion*** *Chair: Olivier Brüls*
12:30–14:00	Lunch
14:00–15:00	Non incremental variational principles for dynamical dissipative systems ***Géry de Saxcé*** *Chair: Simon Eugster*
15:00–15:45	Coffee Break
15:45-17:00	Current Challenges and Emerging Directions in Nonsmooth Dynamics ***Round Table Discussion*** *Chair: Vincent Acary*
18:30	Conference Dinner

The conference dinner will be held at the restaurant “Kitzmann Brauschänke” (Südliche Stadtmauerstraße 25, 91054 Erlangen).

Thursday, October 2nd

09:00–10:00	Discrete-time multivalued port-Hamiltoninan systems with sliding motions ***Félix A. Miranda-Villatoro*** *Chair: Simon Eugster*
10:00–10:30	Robust synchronization of heterogeneous networks using set-valued linear complementarity couplings ***Quang Hung Pham*** *Chair: Olivier Brüls*
10:30–11:00	Coffee Break
11:00–12:00	Grazing Linear Mode, instability, ergodicity for d-degree-of-freedom conservative vibro-impact systems ***Stéphane Junca*** *Chair: Remco Leine*
12:00–12:10	Closing remarks
12:10–13:30	Lunch

List of Abstracts

Tuesday, September 30th

09:45–10:45 — Didactic Introduction

Nonsmooth Dynamics: A Journey from Convex Analysis to Computer Code

Remco Leine

Institute for Nonlinear Mechanics, University of Stuttgart, Germany

Over the past three decades, there has been a growing interest in dynamical systems characterized by non-differentiability, switches, and discontinuities. This surge has given rise to a novel research domain known as Nonsmooth Dynamics, finding diverse applications such as sliding mode control theory, switched electronic circuits, plasticity theory and multibody systems with friction and impact. Mechanics, as one of the oldest natural sciences, has played a forerunner role in formalizing a theory to describe evolution problems involving varying degrees of nonsmoothness.

Traditionally, engineering sciences and physics have mostly adhered to a classical modeling approach, expressing all relationships within a system through explicit equalities; a dogma reinforced with the advent of computers. A crucial point in dealing with nonsmoothness is to leave the equality dogma and allow ourselves to think and work with (variational) inequalities and inclusions. The key merit of modern Nonsmooth Dynamics is the development of a mathematical framework based on convex analysis that effectively describes nonsmooth or set-valued relations within a system. This mathematical framework facilitates a comprehensive understanding of unilateral and nonsmooth behavior, unraveling the intrinsic structure of nonsmooth systems, and allows to devise numerical integration methods for simulating them – prerequisites for the analysis of the nonlinear dynamics of nonsmooth systems.

In this talk, we will embark on a journey through Nonsmooth Dynamics, commencing with fundamental concepts from convex analysis and showing how, in the end, this leads to very effective algorithms for the simulation of nonsmooth systems.

11:30–12:30 — Didactic Introduction

A non-non-smooth model for wheel-rail contact

Martin Arnold

Martin Luther University Halle-Wittenberg, Germany

Non-smooth phenomena in system dynamics result typically from just a few components of a complex technical system. Nevertheless, the corresponding non-smooth component models may deteriorate efficiency and accuracy of classical solvers for the overall system. Specialized solvers for components like tires and for friction force computation in wheel-rail contact are, e.g., a quasi-standard in the simulation of vehicle system dynamics. At system level, such component models with embedded solvers may be considered by co-simulation approaches. Furthermore, component models may include switching functions for handling non-smooth effects efficiently by event capturing. In the first part of the talk, we will summarize and recall some of these well established simulation techniques that support a reliable, robust and efficient dynamical simulation of complex technical systems. In the second part of the talk, one specific problem will be analysed that can not be handled correctly by such standard techniques: the contact between wheel and rail in the dynamical simulation of rail vehicles. A rigid body contact model seems to be appropriate for modelling the contact in normal direction. For wheels with so called wear profiles, the tread's surface is, however, nearly parallel to the track surface. In the contact patch, the distance between both surfaces is nearly constant and minimal changes of the configuration cause jumps of the contact point (that is defined as the point of minimal distance between the two surfaces). The resulting non-smooth behaviour is an artefact of the assumed rigid body contact and can not be observed in measured data from practical experiments. The inappropriate non-smoothness of the mathematical model may be avoided by virtually modified wheel profiles or considering a weighted average of the distance between wheel and rail surfaces.

14:00–15:00 — Moderated Open Discussion

On the accelerated solution of stiff nonsmooth mechanical systems

Jonas Breuling

Institute for Nonlinear Mechanics, University of Stuttgart, Germany

Inspired by the computer graphics community this talk extends Moreau's celebrated midpoint rule for the numerical time integration of nonsmooth mechanical systems with frictional contact and impacts to encompass ideal plasticity and stiff compliance-based force laws. In the limit of infinite stiffness, this framework naturally enforces perfect bilateral constraints at the position level, giving rise to a system of index-three differential-algebraic equations. The proposed extension removes the need for prohibitively small time steps in the presence of very stiff forces, while preserving an accurate treatment of set-valued force laws for contact, friction, and plasticity—without incurring significant computational overhead.

To solve the arising nonlinear normal cone inclusions numerically semi-smooth Newton methods are frequently applied due to their outstanding local convergence rates. A central challenge arises from redundant contacts, frequently encountered in rigid-body simulations, which render the resulting iteration matrix rank-deficient and non-invertible. To address this issue, an inexact Newton method is applied only to the smooth parts of the system, while the set-valued laws are handled through lightweight fixed-point iterations.

Numerical experiments are embedded into the talk to illustrate the validity of the proposed formulation. Moreover, they demonstrate the weak performance of the simple fixed-point iterations, which finally motivates a discussion on three central research questions:

1. How can the local frictional contact solver be accelerated? Can we utilize convex optimization? Are there accelerated variational inequality solvers?

2. Can we leverage nonsmooth contact dynamics to meet industrial standards? Can we derive error measures for adaptive step size control? Can we develop true higher-order methods?

3. What is the broader impact of our research on future applications and its relevance for the next generation? Where do we target the united nations sustainable development goals?

15:30–16:30 — Didactic Introduction

Event-driven Dynamics of Soft-sphere Systems: Bridging Newtonian Mechanics and Non-Smooth Dynamics

Thorsten Pöschel, *Patric Müller*

Institute for Multiscale Simulation, FAU Erlangen-Nürnberg, Germany

The dynamics of dissipative soft-sphere systems are fundamentally governed by Newton's equations of motion. In computational practice, these equations are most commonly solved using force-based Molecular Dynamics schemes, which explicitly integrate Newton's equations to obtain the particle trajectories under the influence of prescribed contact laws and external forces. While accurate and versatile, such schemes are computationally demanding, particularly when simulating large assemblies of particles over long timescales.

Non-smooth event-driven algorithms provide a widely used alternative. These methods exploit the assumption of instantaneous, pairwise collisions, thereby avoiding the need for continuous-time integration of contact forces. Instead, the dynamics are advanced from one collision event to the next, which can drastically accelerate simulations. In this framework, the coefficient of restitution is introduced as a central parameter that must be derived from the underlying particle interaction law.

Despite their efficiency, non-smooth approaches face severe limitations and may fail dramatically. It can be demonstrated that, in certain cases, they yield trajectories that deviate substantially from those predicted by Newton's equations of motion. Vice versa, there are cases where the solution of Newton's equations of motion corresponds to non-plausible (even negative) values of the coefficient of restitution. These observations highlight a fundamental incompatibility between naîve concepts of non-smooth dynamics and Newton's mechanics.

To overcome this conceptual and practical difficulty, we introduce a generalized formulation of the restitution coefficient. This framework provides a mathematically consistent bridge between force-based Molecular Dynamics and non-smooth event-driven simulations. Building on this concept, we derive a numerical scheme which, for dilute systems with frictionless interactions, allows for efficient non-smooth event-driven simulations that remain valid even for non-instantaneous collisions.

Under the generalized restitution framework, particle trajectories obtained from event-driven simulations are in excellent agreement with those from integrating Newton's equation, except during a short transient interval that corresponds to the contact duration. When applied to many-particle systems, the generalized event-driven algorithm retains the computational advantages of event-driven methods while reproducing the physical fidelity of force-based Molecular Dynamics.

Wednesday, October 1st

09:00–10:00 — Didactic Introduction

Semi-resolved FEM-PFEM-DEM models for complex immersed granular free surfaces flows

Vincent Legat, *J. Lambrechts, M. Henry, S. Yans*

Institute of Mechanics, Materials and Civil Engineering, Ecole Polytechnique de Louvain, Belgium

Immersed granular flows, which are mixtures of solid particles and interstitial fluid, are found in a wide range of natural and industrial processes, including landslides and sediment transport and fluidised beds. Modelling them is particularly challenging due to the complex interplay between solid contacts, fluid dynamics, and evolving interfaces. These complex systems are usually modelled using the CFD-DEM approach. Numerical simulation is a powerful way of exploring such systems, providing detailed, time-resolved insights that are often difficult or impossible to capture experimentally. It enables key physical mechanisms to be isolated, hypotheses to be tested, and extreme conditions to be examined, making it an valuable complement to theoretical analysis and experimental work.

Our contribution focuses on the modelling of immersed granular flows across multiple regimes and scales. It begins with the challenge of simulating grains comparable in size to fluid discretisation of the Navier–Stokes equations using an overlap-wise grain representation and semi-implicit time integration. These extensions enable simulation from coarse to fine discretisation and restore the flexibility of the Finite Element Method for handling complex geometries without mesh size restrictions.

The model is extended further to include heat transfer, for which a new correlation is proposed for forced convection in granular suspensions. Spontaneous digging by a water droplet in a hot granular bed is investigated to reveal the main physical mechanisms involved. The model successfully reproduces the key phases of the digging process observed in experiments. As the fluid is modelled using a volume-averaged approach, correlations are introduced to represent the momentum and heat transfer between the grains and the fluid. Firstly, based on the Reynolds analogy, a Nusselt correlation is proposed to quantify the heat transfer of an assembly of grains with a fluid. The proposed law is validated by experimental measurements of heated granular beds taken from the literature. Secondly, the numerical model is validated on the study of a bubbling fluidised granular bed. Finally, the spontaneous excavation of a drop of water in a hot granular bed is studied numerically by the model. When a drop of water is placed on a hot granular bed, depending on the temperature of the bed, it can dig into the bed. The main trends of the experiment, namely sinking, local fluidisation and the formation of a chimney, are reproduced by the simplified two-dimensional numerical model. Heat transfer during the excavation process is investigated to highlight efficient transfer due to the local fluidisation. It is demonstrated that both injection fluidisation and local vaporisation fluidisation are correctly captured, as is heat transfer between the different phases.

10:00–10:30 — PhD talk

A Non-Smooth Model for the Lateral Dynamics of Drill Bits

Kaidong Chen, *Emmanuel Detournay, Nathan van de Wouw*

Eindhoven University of Technology, Netherlands

Drilling is an essential technology for extracting natural resources that are deeply buried in the subsurface. In deep drilling, the drill bit at the end of the slender drill-string can undergo severe self-excited vibrations. Among these vibrations, the lateral vibration of the bit, particularly backward whirl, is recognized as the most damaging mode for polycrystalline diamond compact (PDC) bits.

In this presentation, we introduce a non-smooth model for the lateral dynamics of drill bits. Particular attention is given to the bit-rock interface law, linking the drilling forces on the bit with its penetration into the rock and its velocity. The proposed law is formulated in terms of integrals of non-smooth set-valued functions. How to rigorously determine the normal cone inclusions associated with this constitutive law remains an open question. Under specific assumptions, however, the law can be simplified to integrals of non-smooth single-valued functions.

Simulations based on the simplified bit-rock interaction law show that even a symmetric and well-balanced drill bit can undergo lateral instability through the coupling between the bit-rock interaction and the drill-string compliance. Simulations further suggest that the non-smooth contact and impact between the bit gauge and the wellbore wall play a key role in constraining the amplitude of lateral vibrations, in agreement with both laboratory experiments and field observations.

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11:15–12:30

The ENNSD community

Round Table Discussion

Chaired by Olivier Brüls

This round-table discussion will focus on the activities of the ENNSD community, with the aim of strengthening the network through new formats of interaction. After a brief presentation of the current status, we will exchange ideas about the organisation of future events (frequency, framework, format of talks), including possible online activities such as seminars and communication channels. We will also discuss ways to attract new researchers, foster tighter collaborations across existing codes and approaches, and expand ENNSD's visibility within the broader scientific landscape. Potential opportunities for collaborative funding (such as a COST Action) will be explored. This interactive session aims to gather ideas and concrete proposals that will help shape the next steps of our community.

14:00–15:00 — Didactic Introduction

Non incremental variational principles for dynamical dissipative systems

Géry de Saxcé

University Lille, CNRS, Centrale Lille, UMR 9013 - LaMcube - Laboratoire de mécanique multiphysique multiéchelle, Lille, France

Using the concept of symplectic subdifferential, we propose a modification of the Hamiltonian formalism for dynamical dissipative systems and a space-time variational principle of minimum, called symplectic Brezis-Ekeland-Nayroles principle.

On this basis, we develop new numerical methods downstream using the finite element method to compute the whole loading history in contrast to incremental or step-by-step methods now widely used. We verified the feasibility of these new numerical methods in plasticity and viscoplasticity for tubes and plates in statics and dynamics.

There is a class of constitutive laws, called non associated, which escape from attempts of getting them into the mold of the approach by convex potentials. For them, the bipotential concept allows extending the classical variational principles of minimum in a simple way. We show how to generalize to this class of laws the symplectic Brezis-Ekeland-Nayroles principle thanks to the concept of symplectic bipotential.

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15:45–17:00

Current Challenges and Emerging Directions in Nonsmooth Dynamics

Round Table Discussion

Chaired by Vincent Acary

This session aims to discuss the current challenges and emerging directions in non-smooth dynamics, as well as identifying subjects that can be considered almost complete. The focus will be on pinpointing the key questions driving the field forward. The meeting will foster dialogue across disciplines, highlight hot topics that are shaping the future of non-smooth dynamics and outline collaborative opportunities for addressing urgent theoretical and practical problems.

Thursday, October 2nd

09:00–10:00 — Didactic Introduction

Discrete-time multivalued port-Hamiltoninan systems with sliding motions

Félix A. Miranda-Villatoro

University Grenoble-Alpes, INRIA, CNRS, Grenoble, France

Hamiltonian dynamical systems cover a wide range of applications in diverse fields such as: mechanics, circuit theory, thermodynamics, ecology, and economy, to name a few. In the classical formulation, a Hamiltonian system is characterized by two objects: a symplectic form and a smooth function (called a Hamiltonian function) [1]. The main property of Hamiltonian systems is the simultaneous preservation of the symplectic form and the Hamiltonian function along the system trajectories [2]. For electrical or mechanical systems, the Hamiltonian function corresponds to the energy stored in the system, so preservation of the Hamiltonian amounts to energy conservation.

In this talk we consider a particular generalization of the Hamiltonian approach, suited for control purposes, called port-Hamiltonian systems [4]. This class consists of Hamiltonian systems extended with dissipative elements and external ports so that they can interact with the environment. Port-Hamiltonian systems are not necessarily energy-conserving, but satisfy the more general property of passivity and are amenable to passivity-based control [5]. We focus on discrete-time systems with possibly nonsmooth Hamiltonian functions and nonsmooth Rayleigh dissipation potentials. Our main motivation is that this class of systems contains some higher-order sliding-mode controllers, such as the second-order nested, twisting, and super-twisting algorithms [3]. Our main objective is to derive well-posed discretization schemes that preserve finite-time stability and show robustness against the presence of matched disturbances.

References

[1] V.I. Arnold. *Mathematical Methods of Classical Mechanics*. Springer-Verlag, New York, 1989.

[2] E. Hairer, C. Lubich, and G. Wanner. *Geometric Numerical Integration: Structure-preserving algorithms for ordinary differential equations*. Springer, second edition, 2006.

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10:00–10:30 — PhD talk

Robust synchronization of heterogeneous networks using set-valued linear complementarity couplings

Quang Hung Pham, *Félix A. Miranda-Villatoro*, *Bernard Brogliato*

University Grenoble-Alpes, INRIA, CNRS, Grenoble, France

The synchronization of heterogeneous networks is an open problem that has attracted significant attention from the systems and control community over the past decade. Due to the intrinsic complexity of such networks, most previous studies have primarily focused on so-called practical synchronization rather than asymptotic synchronization. This work investigates a heterogeneous network of time-varying nonlinear dynamical systems, and proposes a new class of couplings to achieve global exponential (finite-time) synchronization. These couplings are designed as linear complementarity constraints (and hence may be set-valued), form a direct extension of conventional linear couplings. Under the proposed couplings, the network becomes a closed-loop system, taking the form of linear complementarity systems with a specific structure. Closed-loop well-posedness, exponential and finite-time stability, and robustness with respect to uncertainties and matched perturbations are studied. Passivity and maximal monotonicity of operators are the main analysis tools. Algorithms enabling the design of the discrete-time controllers for practical implementation, and an example of circuits synchronization illustrating the theoretical findings, are provided. It is shown that the proposed framework allows to consider new nonsmooth nonlinear feedback controllers. This is crucial for practical applications.

Grazing Linear Mode, instability, ergodicity, for d-degree-of-freedom conservative vibro-impact systems

Stéphane Junca [1], *Mathias Legrand* [2], *Huong Le Thi* [3], *Vivien Tirel* [4]

[1] Mathematiques, Université Côte d'Azur, Nice, France
[2] Mécanique, Ecole Centrale de Nantes, France
[3] Mathematics, Tang Long Univ., Hanoi, Vietnam
[4] Physique, Université Côte d'Azur, Nice, France

For conservative vibro-impact systems, the instability of Grazing Linear Mode is conjectured since at least Nordmark [5] using a First Return Map (FRM) [1] and a square-root singularity (SQRS) [6]. Another singular FRM is compared to the first one. Our FRM has already provided closed forms for Non Smooth Modes with one impact per period (1IPP) [3] and details a complex decomposition of the contact hyperplane with infinitely many SQRS [4] reflecting the non smooth dynamics near a GLM. Ergodicity of this dynamics is questioned [2].

References

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